# Asset Allocation with Option-implied distributions

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# Outline

- Motivation
- Implied distributions: Definition and Literature
- Dataset and risk-adjusting option-implied distributions
- Results
- Conclusions and suggestions for professional applications

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- Modern approach: Maximize expected utility over next period wealth
- Two ways to maximize expected utility:

1. Direct utility maximization (Adler & Kritzman, 2007, Sharpe, 2007)
2. Approximate expected utility via Taylor series expansion (Levy & Markowitz, 1979, Jondeau & Rockinger, 2006)

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- Our intuition: Utilize forward-looking information obtained from option markets to construct portfolios

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- Out-of-sample performance calculated under various metrics
- Performance of these portfolios is *compared* with the performance of portfolios formed using *purely* historical dbns

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- Various measures of portfolio performance

# Implied distributions: Definition

- Breeden and Litzenberger (1978): Extract the pdf for underlying asset, $f(S_T)$, from call option prices at different strike prices via:

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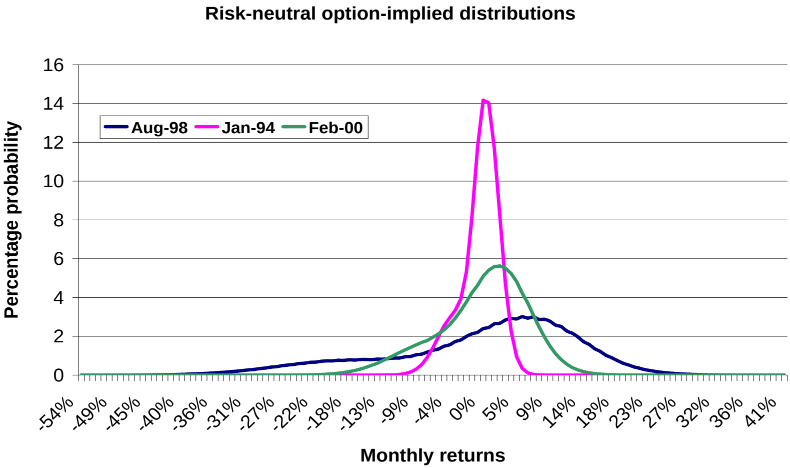
- Having a continuum of strike prices, get a risk-neutral option-implied pdf for the underlying asset

# Implied distributions: Definition

- Implied dbns exhibit considerable time-variation-> may contain valuable information over future returns



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- Related study: Ait-Sahalia and Brandt (2008, NBER) utilize risk-neutral implied dbns for asset allocation

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- Obtain risk-adjusted implied dbns via Bliss-Panigirtzoglou (2004, JF) method
- Estimate historical dbns via a Gaussian kernel (Jackwerth, 2000)

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- Fit a natural spline to obtain $g(\delta)$ & get 20,000 points to convert them back to the price/strike space
- Differentiate numerically this call pricing function wrt different strikes to obtain risk-neutral pdfs.

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2. Test forecasting ability of the transformed (risk-adjusted) implied pdfs via Berkowitz (2001) methodology
3. Perform appropriate risk-adjustment to maximize p-value of Berkowitz LR test (i.e. maximize forecasting ability)

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$$q_t(S_T) = \zeta(S_T) \times p_t(S_T)$$

- where

$$\zeta(S_T) \equiv \exp[r(T-t)]\frac{U'(S_T)}{U'(S_t)}$$

is the pricing kernel (ratio of marginal utilities)

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$$U(W) = \frac{W^{1-\gamma}}{1-\gamma}, \ \ \gamma \neq 1$$

- Exponential utility with ARA coefficient $\eta$:

$$U(W) = -\exp(-\eta W)/\eta, \ \ \eta \neq 0$$

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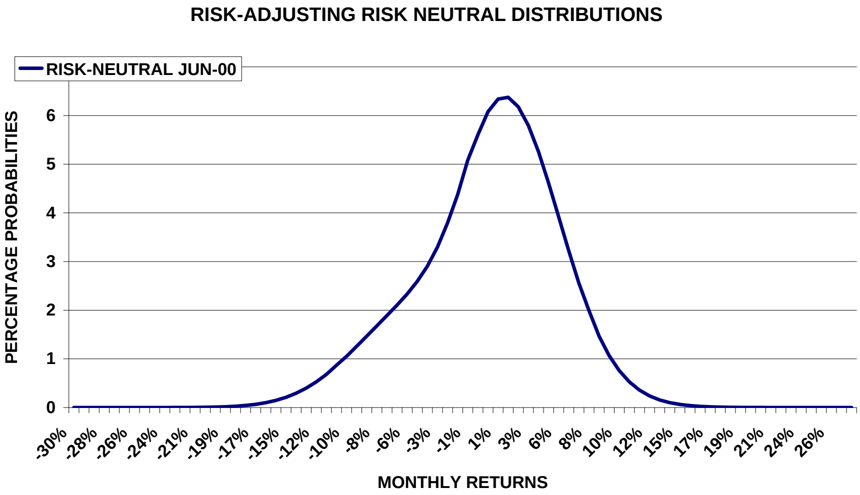
- Aim: find $\gamma^*(\eta^*)$ that maximizes p-value of Berkowitz LR test
- Use optimal $\gamma^*(\eta^*)$ for realizations up to $t$ to risk-adjust the RN implied dbn at $t$ over $t+1$ (one-month maturity)
- i.e. use information (RN dbns, realizations and implied $\gamma^*$or $\eta^*$) known only up to $t$ -> out-of-sample exercise

# Risk-adjusting the implied distributions



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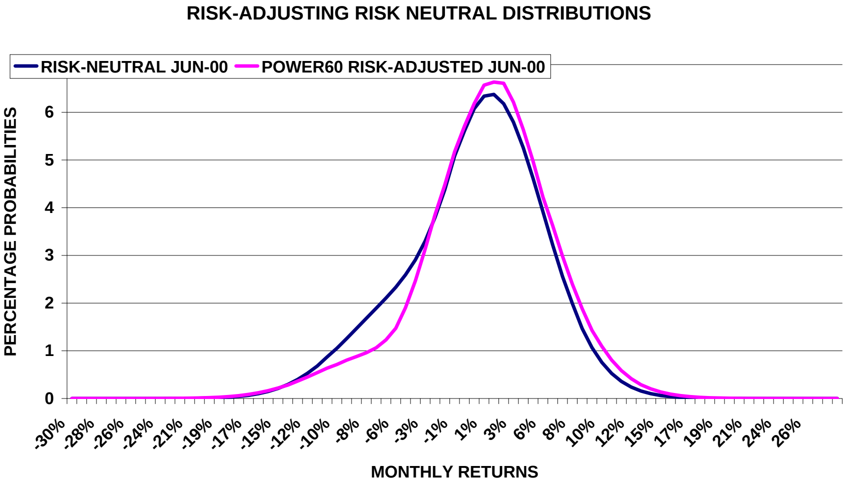
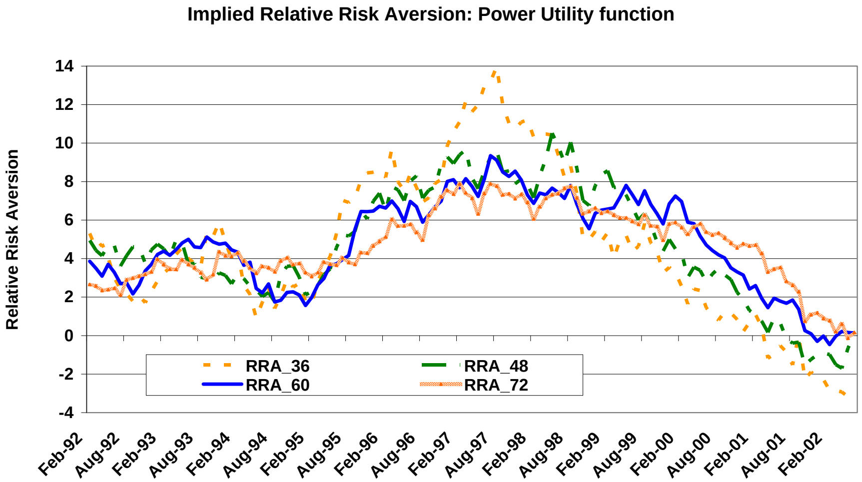
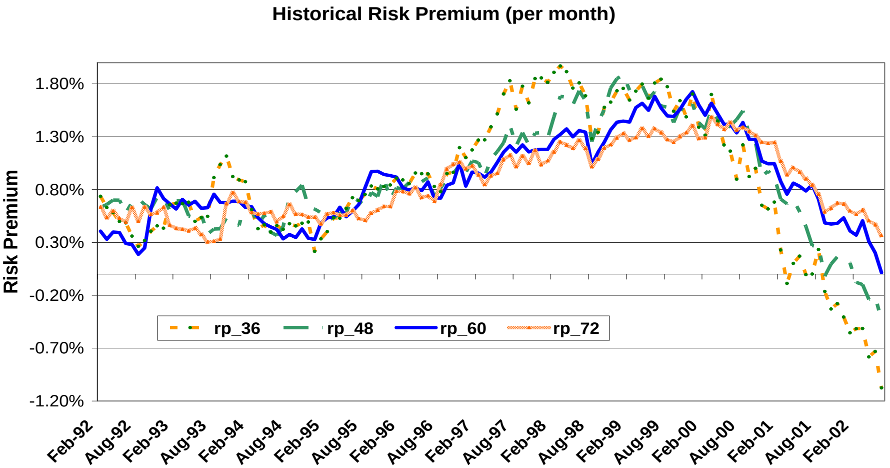


# Implied RRA: Power utility function



# S&P 500 Historical Risk premium (per month)



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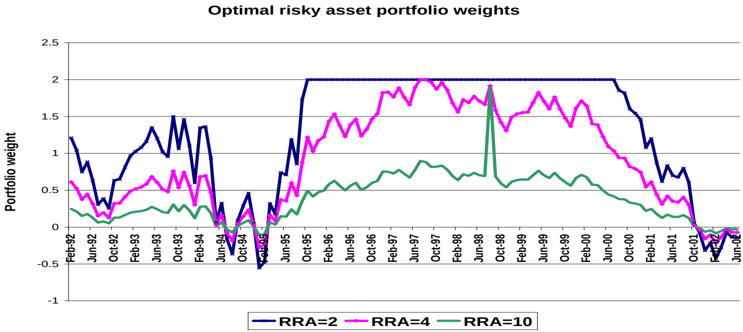
- Alternatively use power & exponential utility for individual investor

- Also perform moments-based optimizations via truncated Taylor series expansions up to 4th order

# Calculating optimal portfolios: Direct power utility

- Restrict optimal weights $\alpha_t \in [-1, 2]$ to prevent extreme weights from driving our results



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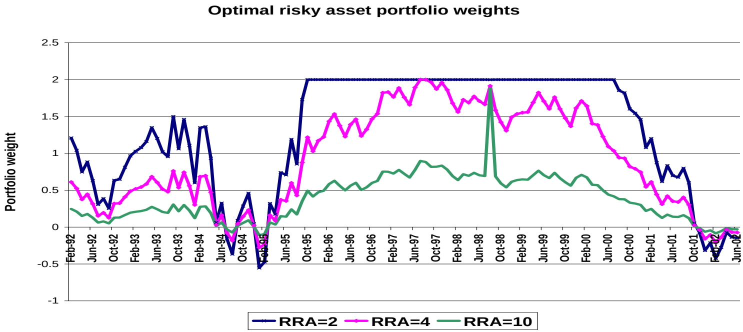


- Similar evolution for weights using only historical distributions

# Sharpe Ratios: Direct maximization with power utility (03/1992-06/2002)

| | RRA=2 | RRA=4 | RRA=6 | RRA=8 | RRA=10 |
|---|---|---|---|---|---|
| **Panel A: Risk-Adjusted Implied Distributions** | | | | | |
| SR_36 | **0.75** | **0.71** | **0.72** | **0.73** | **0.73** |
| SR_48 | **0.64** | **0.6** | **0.61** | **0.62** | **0.63** |
| SR _60 | **0.46** | **0.5** | **0.51** | **0.53** | **0.54** |
| SR _72 | **0.37** | **0.39** | **0.41** | **0.43** | **0.44** |
| **Panel B: Historical Distributions** | | | | | |
| SR_36 | **0.56** | **0.52** | **0.5** | **0.48** | **0.48** |
| SR_48 | **0.47** | **0.49** | **0.36** | **0.36** | **0.36** |
| SR _60 | **0.38** | **0.4** | **0.35** | **0.35** | **0.35** |
| SR _72 | **0.28** | **0.27** | **0.24** | **0.24** | **0.24** |

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- Note: Naive 1/N strategy yields SR=0.33 in our sample
- p-values show significant difference for most of the cases

# SRs: 2nd order Taylor series expansion of power utility

| Panel A: Risk-Adjusted Implied Distributions | | | | | |
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| SR_36 | **0.73** | **0.73** | **0.66** | **0.68** | **0.69** |
| SR_48 | **0.62** | **0.64** | **0.58** | **0.55** | **0.59** |
| SR _60 | **0.43** | **0.52** | **0.49** | **0.54** | **0.57** |
| SR _72 | **0.36** | **0.41** | **0.39** | **0.44** | **0.47** |
| **Panel B: Historical Distributions** | | | | | |
| SR_36 | **0.57** | **0.54** | **0.47** | **0.50** | **0.48** |
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| SR _60 | **0.35** | **0.43** | **0.33** | **0.34** | **0.34** |
| SR _72 | **0.27** | **0.27** | **0.22** | **0.22** | **0.22** |

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$$E[U(1 + r_p^{hist} + c)] = E[U(1 + r_p^{imp})]$$

- Different from mean-variance measures because expected utility is affected by higher moments too

# Opportunity cost: Direct maximization of power and exponential utility

| Panel A: Power Utility function | | | | | |
|---|---|---|---|---|---|
| | RRA=2 | RRA=4 | RRA=6 | RRA=8 | RRA=10 |
| 36_Obs | **3.72%** | **3.84%** | **4.80%** | **3.96%** | **8.04%** |
| 48_Obs | **3.84%** | **2.40%** | **6.84%** | **5.04%** | **3.96%** |
| 60_Obs | **2.16%** | **2.88%** | **3.36%** | **2.64%** | **2.16%** |
| 72_Obs | **2.28%** | **3.24%** | **3.24%** | **2.52%** | **2.04%** |
| **Panel B: Exponential Utility function** | | | | | |
| | ARA=2 | ARA=4 | ARA=6 | ARA=8 | ARA=10 |
| 36_Obs | **3.12%** | **3.36%** | **1.56%** | **1.56%** | **1.20%** |
| 48_Obs | **3.12%** | **2.16%** | **3.48%** | **2.64%** | **2.04%** |
| 60_Obs | **2.40%** | **1.44%** | **-0.60%** | **-0.48%** | **-0.36%** |
| 72_Obs | **3.00%** | **1.92%** | **-0.48%** | **-0.36%** | **-0.24%** |

# Portfolio Turnover

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- Calculate portfolio turnover implied by our approach & compare it to the turnover of purely historical strategy
- Turnover of strategy $k$ involving $N$ assets & $T$ rebalancing points:

$$PT_k = \frac{1}{T-1}\sum_{t=1}^{T}\sum_{j=1}^{N}\left(\left|w_{k,j,t+1} - w_{k,j,t^+}\right|\right)$$

where $w_{k,j,t+1}$: optimal portfolio weight at $t+1$

$w_{k,j,t^+}$: existing portfolio weight before rebalancing at $t+1$

# Portfolio Turnover: Direct utility maximization

- Turnover ratios of portfolios formed using risk-adjusted implied dbns wrt portfolios formed using historical dbns

| Panel A: Turnover Ratio for Power utility | | | | | |
|---|---|---|---|---|---|
| | RRA=2 | RRA=4 | RRA=6 | RRA=8 | RRA=10 |
| ratio _36 | **1.17** | **1.02** | **0.97** | **1.03** | **1.14** |
| ratio _48 | **1.31** | **1.21** | **1.06** | **1.27** | **1.44** |
| ratio _60 | **1.41** | **1.12** | **1.20** | **1.42** | **1.62** |
| ratio_72 | **1.46** | **0.99** | **1.17** | **1.41** | **1.63** |
| **Panel B: Turnover Ratio for exponential utility** | | | | | |
| | ARA=2 | ARA=4 | ARA=6 | ARA=8 | ARA=10 |
| ratio_36 | **1.08** | **0.91** | **0.78** | **0.77** | **0.82** |
| ratio _48 | **1.12** | **1.00** | **0.91** | **0.96** | **1.02** |
| ratio _60 | **1.40** | **1.03** | **1.17** | **1.25** | **1.33** |
| ratio _72 | **1.37** | **1.01** | **1.15** | **1.22** | **1.30** |

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| Panel A: Turnover Ratio for Power utility | | | | | |
|---|---|---|---|---|---|
| | RRA=2 | RRA=4 | RRA=6 | RRA=8 | RRA=10 |
| ratio _36 | **1.17** | **1.02** | **0.97** | **1.03** | **1.14** |
| ratio _48 | **1.31** | **1.21** | **1.06** | **1.27** | **1.44** |
| ratio _60 | **1.41** | **1.12** | **1.20** | **1.42** | **1.62** |
| ratio_72 | **1.46** | **0.99** | **1.17** | **1.41** | **1.63** |
| **Panel B: Turnover Ratio for exponential utility** | | | | | |
| | ARA=2 | ARA=4 | ARA=6 | ARA=8 | ARA=10 |
| ratio_36 | **1.08** | **0.91** | **0.78** | **0.77** | **0.82** |
| ratio _48 | **1.12** | **1.00** | **0.91** | **0.96** | **1.02** |
| ratio _60 | **1.40** | **1.03** | **1.17** | **1.25** | **1.33** |
| ratio _72 | **1.37** | **1.01** | **1.15** | **1.22** | **1.30** |

- Very similar results for 2nd-4th order Taylor series expansions

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- Using returns net of transaction costs, returns-loss metric given by:

$$return - loss = \frac{\mu_{imp}}{\sigma_{imp}} \times \sigma_{hist} - \mu_{hist}$$

# Returns-loss metric: Direct utility maximization

- Excess risk-adjusted returns derived from implied dbns strategy net of transaction costs

| Panel A: Return-Loss for Power Utility case | | | | | |
|---|---|---|---|---|---|
| | RRA=2 | RRA=4 | RRA=6 | RRA=8 | RRA=10 |
| return-loss_36 | **3.90%** | **3.60%** | **3.53%** | **2.97%** | **2.39%** |
| return-loss_48 | **3.64%** | **2.12%** | **3.76%** | **2.87%** | **2.32%** |
| return-loss_60 | **1.67%** | **1.73%** | **2.07%** | **1.64%** | **1.37%** |
| return-loss_72 | **1.72%** | **2.27%** | **2.21%** | **1.76%** | **1.48%** |
| **Panel B: Return-Loss for Exponential Utility case** | | | | | |
| | ARA=2 | ARA=4 | ARA=6 | ARA=8 | ARA=10 |
| return-loss_36 | **3.33%** | **2.83%** | **1.59%** | **1.42%** | **1.10%** |
| return-loss_48 | **3.10%** | **2.21%** | **2.90%** | **2.16%** | **1.71%** |
| return-loss_60 | **2.00%** | **1.81%** | **1.23%** | **0.91%** | **0.70%** |
| return-loss_72 | **2.65%** | **3.02%** | **2.20%** | **1.63%** | **1.29%** |

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- Proposed a methodology using risk-adjusted option-implied dbns
- Out-of-sample performance of our approach is compared with performance of a purely historical approach
- Forward-looking approach makes the investor better off even when transaction costs are taken into account
- Value and scope in exploiting information embedded in option-implied dbns for asset allocation purposes

# Suggestions for future research

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2. Alternative methods to extract implied dbns
3. Alternative methods & utility functions to risk-adjust implied pdfs
4. Form portfolios with multiple risky assets (use baskets of options or copula methods to get joint pdfs)

# Thank you for attending!