

# Higher co-moments and asset pricing on London Stock Exchange

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# Presentation Outline

- Motivation: Life beyond mean and variance
- Co-moments and asset pricing: Theory
- Co-moments and asset pricing: Evidence from LSE
- Current research and conclusions

# Motivation: Failure of the CAPM

- Vast literature documenting the empirical failure of the CAPM
- Long list of "anomalies" / strategies that generate large pricing errors / alphas (Fama and French, 2008)
- Two-period model: Important limitation when returns are not i.i.d. (intertemporal premia: Merton's ICAPM)
- Mean-variance model: Important limitation when asset returns are not symmetric and preferences are not quadratic (higher co-moments premia)

# There is life beyond mean and variance

- Asset returns exhibit negative skewness and excess kurtosis (e.g. financial distress, bankruptcy, bond defaults)
- In the presence of stochastically time-varying risk factors unconditional returns' distributions are non-normal (e.g. stochastic volatility model of Heston, 1992)
- Utility theory accommodates preferences over skewness and kurtosis
- Prudence (Kimball, 1990): Expect  $U^{(3)} > 0$
- Temperance (Eeckhoudt et al., 1996): Expect  $U^{(4)} < 0$
- Related: First-order risk aversion (Segal and Spivak, 1990) and Loss Aversion (Kahnemann and Tversky, 1979)

# Ample evidence for preferences over skewness and kurtosis

- Investors dislike negative skewness and excess kurtosis in payoff distributions
- Examples: Expensive insurance products, overpriced out-of-the-money puts, capital-guarantee products
- Modern risk management: Value-at-risk and portfolio insurance
- Investors like positive skewness in payoff distributions
- Examples: Participation in unfair games with positively skewed payoffs (jackpots in lotteries and premium bonds)

- Preferences over higher moments in asset pricing: Kraus and Litzenberger (1976) and Scott and Horvath (1980)
- Empirical evidence for pricing of coskewness/ cokurtosis risk: Harvey and Siddique (2000), Dittmar (2002) for US, only Hung, Shackleton and Xu (2004) for the UK
- Higher co-moments models for performance evaluation: Moreno and Rodriguez (2009) for US and Kostakis (2009) for UK
- Limitations of the higher co-moments model: Dittmar (2002), Post et al. (2008), Poti and Wang (2010)

- Examine whether coskewness and cokurtosis risks are priced on LSE
- Show that standard asset pricing theory predicts an extra premium is required for shares exhibiting negative coskewness and positive cokurtosis with the market portfolio
- Estimate CSK and CKT for a large cross section of LSE-listed shares on a monthly basis
- Use CSK and CKT as sorting criteria to construct decile portfolios
- Examine their post-ranking returns and perform standard asset pricing tests

Starting from the central asset pricing equation,  $p_t = E(m_{t+1}x_{t+1})$ , the premium for any financial asset is given by:

$$E_t(r_{t+1}) - r_t^f = -(1 + r_t^f) \text{Cov}(m_{t+1}, r_{t+1})$$

Common choice for SDF:  $m_{t+1} = \frac{U'(W_{t+1})}{U'(W_t)}$

Taking a 1st order Taylor series expansion of  $U'(W_{t+1})$  around  $W_t$ , the SDF becomes linear in market returns:

$$m_{t+1} = 1 + \tilde{b}r_{m,t+1}$$

This leads to the CAPM and implies the standard result:

$$E_t(r_{t+1}) - r_t^f = -(1 + r_t^f)\tilde{b}\text{Cov}(r_{m,t+1}, r_{t+1})$$



- No reason to truncate the expansion at 1<sup>st</sup> order. Taking the 3<sup>rd</sup> order expansion of  $U'(W_{t+1})$  around  $W_t$ , the SDF becomes:

$$\begin{aligned}\frac{U'(W_{t+1})}{U'(W_t)} &\simeq 1 + \frac{U''(W_t)W_t}{U'(W_t)}r_{m,t+1} + \frac{U^{(3)}(W_t)W_t^2}{2U'(W_t)}r_{m,t+1}^2 + \frac{U^{(4)}(W_t)W_t^3}{6U'(W_t)}r_{m,t+1}^3 \\ &= 1 - \gamma r_{m,t+1} + \frac{1}{2}\gamma\eta r_{m,t+1}^2 - \frac{1}{6}\gamma\eta\kappa r_{m,t+1}^3\end{aligned}$$

- $\eta \equiv -\frac{U^{(3)}(W)W}{U''(W)}$  is Kimball's (1990) measure of relative *prudence*
- $\kappa \equiv -\frac{U^{(4)}(W)W}{U^{(3)}(W)}$  is Eeckhoudt et al. (1996) measure of relative *temperance*.
- We get an SDF which includes the squared and cubic market returns:

$$m_{t+1} = 1 + \tilde{b}r_{m,t+1} + \tilde{c}r_{m,t+1}^2 + \tilde{d}r_{m,t+1}^3$$

This SDF specification implies the following relationship:

$$E_t(r_{t+1}) - r_t^f = -(1 + r_t^f)\tilde{b}\text{Cov}(r_{m,t+1}, r_{t+1}) - (1 + r_t^f)\tilde{c}\text{Cov}(r_{m,t+1}^2, r_{t+1}) - (1 + r_t^f)\tilde{d}\text{Cov}(r_{m,t+1}^3, r_{t+1})$$

Consequently, the risk premium now has three components:

- 1 Covariance premium (Covariance  $\uparrow \Rightarrow$  premium  $\uparrow$ )
- 2 Coskewness premium (Coskewness  $\uparrow \Rightarrow$  premium  $\downarrow$ )
- 3 Cokurtosis premium (Cokurtosis  $\uparrow \Rightarrow$  premium  $\uparrow$ )

# Measures of coskewness and cokurtosis

- Measure of standardized coskewness:

$$CSK_t = \frac{E[\varepsilon_{i,t}\varepsilon_{m,t}^2]}{\sqrt{E[\varepsilon_{i,t}^2]E[\varepsilon_{m,t}^2]}}$$

- Measure of standardized cokurtosis:

$$CKT_t = \frac{E[\varepsilon_{i,t}\varepsilon_{m,t}^3]}{\sqrt{E[\varepsilon_{i,t}^2]E[\varepsilon_{m,t}^3]}}$$

where  $\varepsilon_{i,t}$  is the residual from the CAPM regression for each asset  $i$

- Advantage vs Huang-Litzenberger (1976): These coskewness and cokurtosis measures are orthogonal to beta risk

- Utilize all LSE-listed shares during January 1986- December 2008 (clean sample: 3,501 shares)
- Estimate CSK and CKT measures for each share and each month using a window of 60 monthly returns
- Sort shares each month  $t$  according to CSK (correspondingly CKT) in ascending order
- Construct decile CSK and CKT portfolios and calculate excess returns
- Datastream Reuters, LSPD for delisting reasons, Market, SMB, HML and MOM factors for UK

# Characteristics of coskewness-sorted portfolios

**Characteristics of decile Coskewness portfolios (From Negative CSK to Positive CSK)**

	<b>P1</b>	<b>P2</b>	<b>P9</b>	<b>P10</b>	<b>P1-P10</b>	<b><i>t</i>-test</b>
<b>Average CSK</b>	<b>-0.41</b>	<b>-0.27</b>	<b>0.16</b>	<b>0.34</b>	<b>-0.75</b>	<b>-61.70</b>
<b>EW returns (% p.a)</b>	<b>3.92</b>	<b>3.41</b>	<b>-1.64</b>	<b>-2.71</b>	<b>6.64</b>	<b>2.24</b>
<b>VW returns (% p.a)</b>	<b>6.81</b>	<b>4.59</b>	<b>-0.16</b>	<b>-2.38</b>	<b>9.20</b>	<b>2.07</b>
<b>MV (£m)</b>	<b>956</b>	<b>685</b>	<b>1006</b>	<b>1628</b>	<b>-671</b>	<b>-14.55</b>
<b>CAPM Beta</b>	<b>0.81</b>	<b>0.90</b>	<b>0.94</b>	<b>1.08</b>	<b>-0.28</b>	<b>-8.60</b>

# Characteristics of cokurtosis-sorted portfolios

**Characteristics of decile Cokurtosis portfolios (From Negative CKT to Positive CKT)**

	<b>P1</b>	<b>P2</b>	<b>P9</b>	<b>P10</b>	<b>P10-P1</b>	<i>t-stat</i>
<b>Average CKT</b>	<b>-0.76</b>	<b>-0.43</b>	<b>0.43</b>	<b>0.75</b>	<b>1.52</b>	<b>21.11</b>
<b>EW returns (% p.a)</b>	<b>-4.26</b>	<b>-2.17</b>	<b>2.41</b>	<b>3.36</b>	<b>7.63</b>	<b>3.00</b>
<b>VW returns (% p.a)</b>	<b>-0.58</b>	<b>-0.01</b>	<b>3.29</b>	<b>7.05</b>	<b>7.63</b>	<b>1.87</b>
<b>MV (£m)</b>	<b>1101</b>	<b>771</b>	<b>1056</b>	<b>1381</b>	<b>279</b>	<b>4.17</b>
<b>CAPM Beta</b>	<b>1.08</b>	<b>1.09</b>	<b>0.97</b>	<b>0.80</b>	<b>-0.28</b>	<b>-11.96</b>

# Performance of coskewness-sorted portfolios

Alphas of value-weighted Coskewness portfolios

	P1	P2	P9	P10	P1-P10	Chi-sq.
<b>CAPM alpha (% p.a)</b>	<b>4.60</b>	<b>2.13</b>	<b>-2.74</b>	<b>-5.36</b>	<b>9.96</b>	<b>17.44</b>
	<b>(1.54)</b>	<b>(0.71)</b>	<b>(-1.17)</b>	<b>(-2.07)**</b>	<b>(2.67)***</b>	<b>(0.06)</b>
<b>Fama-French alpha</b>	<b>3.88</b>	<b>2.18</b>	<b>-2.83</b>	<b>-5.92</b>	<b>9.81</b>	<b>19.54</b>
	<b>(1.57)</b>	<b>(0.79)</b>	<b>(-1.20)</b>	<b>(-2.41)**</b>	<b>(2.77)***</b>	<b>(0.03)</b>
<b>Carhart alpha</b>	<b>4.37</b>	<b>2.70</b>	<b>-2.49</b>	<b>-3.38</b>	<b>7.74</b>	<b>22.00</b>
	<b>(1.78)*</b>	<b>(1.09)</b>	<b>(-1.20)</b>	<b>(-1.17)</b>	<b>(2.07)**</b>	<b>(0.02)</b>

# Performance of cokurtosis-sorted portfolios

Alphas of value-weighted Cokurtosis portfolios

	<b>P1</b>	<b>P2</b>	<b>P9</b>	<b>P10</b>	<b>P10-P1</b>	<b>Chi-sq.</b>
<b>CAPM alpha (% p.a)</b>	<b>-3.55</b> (-1.15)	<b>-3.01</b> (-1.09)	<b>0.62</b> (0.21)	<b>4.86</b> (2.10)**	<b>8.41</b> (2.10)**	<b>19.34</b> (0.03)
<b>Fama-French alpha</b>	<b>-4.52</b> (-1.61)*	<b>-3.14</b> (-1.35)	<b>0.50</b> (0.16)	<b>4.09</b> (1.89)**	<b>8.61</b> (2.32)**	<b>31.43</b> (0.00)
<b>Carhart alpha</b>	<b>-2.37</b> (-0.78)	<b>-2.74</b> (-1.08)	<b>0.82</b> (0.29)	<b>5.19</b> (2.49)**	<b>7.57</b> (1.96)**	<b>36.69</b> (0.00)



- 2-step Fama-MacBeth tests for the cross-section of CSK (CKT) portfolios using standard market, size, value and momentum factors
- Main finding: Standard common factors cannot price the cross-section of CSK (or CKT) portfolios' returns
- Construct coskewness and cokurtosis zero-cost (spread) risk factors using a 20% cutoff percentage value
- Augment the CAPM with these two risk factors

$$r_i^e = \alpha_i + \beta_{i,m} r_m^e + \beta_{i,S^- - S^+} (S^- - S^+) + \beta_{i,K^+ - K^-} (K^+ - K^-) + \varepsilon_i$$

- and use augmented higher co-moments model for asset pricing tests
- Evidence that these risk factors can help explain the cross-section of CSK and CKT portfolios (verification)

# Robustness checks and current research

- Equally weighted portfolios
- Different windows of observations for the estimation of CSK and CKT (48 & 72 months)
- Different cutoff percentage values for the construction of the risk factors
- Current research: Utilize the higher co-moments asset pricing model to re-examine the abnormal performance of "anomalies" in the UK
- Preliminary results: Pricing errors are reduced (but not eliminated). Most interesting: Reduction of momentum abnormal profits.

- Higher co-moments are relatively neglected in empirical asset pricing
- Ample evidence that there is life beyond mean and variance
- Strong theoretical foundations in asset pricing and utility theory vs. purely empirical risk factors
- Negative coskewness risk and positive cokurtosis risk are significantly priced in a large cross-section of LSE-listed shares
- Promising area of research for international financial markets, cost of capital, performance evaluation, anomalies studies