Higher co-moments and asset pricing on London Stock Exchange

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Presentation Outline

- Motivation: Life beyond mean and variance
- Co-moments and asset pricing: Theory
- Co-moments and asset pricing: Evidence from LSE
- Current research and conclusions
Motivation: Failure of the CAPM

- Vast literature documenting the empirical failure of the CAPM

- Long list of "anomalies"/strategies that generate large pricing errors/alphas (Fama and French, 2008)

- Two-period model: Important limitation when returns are not i.i.d. (intertemporal premia: Merton’s ICAPM)

- Mean-variance model: Important limitation when asset returns are not symmetric and preferences are not quadratic (higher co-moments premia)
There is life beyond mean and variance

- Asset returns exhibit negative skewness and excess kurtosis (e.g. financial distress, bankruptcy, bond defaults)
- In the presence of stochastically time-varying risk factors unconditional returns’ distributions are non-normal (e.g. stochastic volatility model of Heston, 1992)
- Utility theory accommodates preferences over skewness and kurtosis
  - Prudence (Kimball, 1990): Expect $U^{(3)} > 0$
  - Temperance (Eeckhoudt et al., 1996): Expect $U^{(4)} < 0$
- Related: First-order risk aversion (Segal and Spivak, 1990) and Loss Aversion (Kahnemann and Tversky, 1979)
Investors dislike negative skewness and excess kurtosis in payoff distributions

Examples: Expensive insurance products, overpriced out-of-the-money puts, capital-guarantee products

Modern risk management: Value-at-risk and portfolio insurance

Investors like positive skewness in payoff distributions

Examples: Participation in unfair games with positively skewed payoffs (jackpots in lotteries and premium bonds)
Prior literature

- Preferences over higher moments in asset pricing: Kraus and Litzenberger (1976) and Scott and Horvath (1980)


This study

- Examine whether coskewness and cokurtosis risks are priced on LSE

- Show that standard asset pricing theory predicts an extra premium is required for shares exhibiting negative coskewness and positive cokurtosis with the market portfolio

- Estimate CSK and CKT for a large cross section of LSE-listed shares on a monthly basis

- Use CSK and CKT as sorting criteria to construct decile portfolios

- Examine their post-ranking returns and perform standard asset pricing tests
Starting from the central asset pricing equation, $p_t = E(m_{t+1}x_{t+1})$, the premium for any financial asset is given by:

$$E_t(r_{t+1}) - r^f_t = -(1 + r^f_t) Cov(m_{t+1}, r_{t+1})$$

Common choice for SDF: $m_{t+1} = \frac{U'(W_{t+1})}{U'(W_t)}$

Taking a 1st order Taylor series expansion of $U'(W_{t+1})$ around $W_t$, the SDF becomes linear in market returns:

$$m_{t+1} = 1 + \tilde{b}r_{m,t+1}$$

This leads to the CAPM and implies the standard result:

$$E_t(r_{t+1}) - r^f_t = -(1 + r^f_t)\tilde{b} Cov(r_{m,t+1}, r_{t+1})$$
Asset pricing theory

- No reason to truncate the expansion at 1\textsuperscript{st} order. Taking the 3\textsuperscript{rd} order expansion of $U'(W_{t+1})$ around $W_t$, the SDF becomes:

$$
\frac{U'(W_{t+1})}{U'(W_t)} \approx 1 + \frac{U''(W_t)W_t}{U'(W_t)} r_{m,t+1} + \frac{U^{(3)}(W_t)W_t^2}{2U'(W_t)} r_{m,t+1}^2 + \frac{U^{(4)}(W_t)W_t^3}{6U'(W_t)} r_{m,t+1}^3
$$

$$
= 1 - \gamma r_{m,t+1} + \frac{1}{2} \gamma \eta r_{m,t+1}^2 - \frac{1}{6} \gamma \eta \kappa r_{m,t+1}^3
$$

- $\eta \equiv -\frac{U^{(3)}(W)W}{U''(W)}$ is Kimball's (1990) measure of relative prudence

- $\kappa \equiv -\frac{U^{(4)}(W)W}{U^{(3)}(W)}$ is Eeckhoudt et al. (1996) measure of relative temperance.

- We get an SDF which includes the squared and cubic market returns:

$$
m_{t+1} = 1 + \tilde{b} r_{m,t+1} + \tilde{c} r_{m,t+1}^2 + \tilde{d} r_{m,t+1}^3
$$
Asset pricing theory

This SDF specification implies the following relationship:

\[ E_t(r_{t+1}) - r_t^f = -(1 + r_t^f) \tilde{b} \text{Cov}(r_{m,t+1}, r_{t+1}) - (1 + r_t^f) \tilde{c} \text{Cov}(r_{m,t+1}^2, r_{t+1}) - (1 + r_t^f) \tilde{d} \text{Cov}(r_{m,t+1}^3, r_{t+1}) \]

Consequently, the risk premium now has three components:

1. Covariance premium (Covariance \( \uparrow \Rightarrow \) premium \( \uparrow \))
2. Coskewness premium (Coskewness \( \uparrow \Rightarrow \) premium \( \downarrow \))
3. Cokurtosis premium (Cokurtosis \( \uparrow \Rightarrow \) premium \( \uparrow \))
Measures of coskewness and cokurtosis

- Measure of standardized coskewness:
  \[
  CSK_t = \frac{E[\varepsilon_{i,t}\varepsilon_{m,t}^2]}{\sqrt{E[\varepsilon_{i,t}^2]E[\varepsilon_{m,t}^2]}}
  \]

- Measure of standardized cokurtosis:
  \[
  CKT_t = \frac{E[\varepsilon_{i,t}\varepsilon_{m,t}^3]}{\sqrt{E[\varepsilon_{i,t}^2]E[\varepsilon_{m,t}^3]}}
  \]

where \( \varepsilon_{i,t} \) is the residual from the CAPM regression for each asset \( i \)

- Advantage vs Huang-Litzenberger (1976): These coskewness and cokurtosis measures are orthogonal to beta risk
Data and methodology

- Utilize all LSE-listed shares during January 1986- December 2008 (clean sample: 3,501 shares)

- Estimate CSK and CKT measures for each share and each month using a window of 60 monthly returns

- Sort shares each month $t$ according to CSK (correspondingly CKT) in ascending order

- Construct decile CSK and CKT portfolios and calculate excess returns

- Datastream Reuters, LSPD for delisting reasons, Market, SMB, HML and MOM factors for UK
### Characteristics of decile Coskewness portfolios (From Negative CSK to Positive CSK)

<table>
<thead>
<tr>
<th></th>
<th>P1</th>
<th>P2</th>
<th>P9</th>
<th>P10</th>
<th>P1-P10</th>
<th>t-test</th>
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</thead>
<tbody>
<tr>
<td>Average CSK</td>
<td>-0.41</td>
<td>-0.27</td>
<td>0.16</td>
<td>0.34</td>
<td>-0.75</td>
<td>-61.70</td>
</tr>
<tr>
<td>EW returns (% p.a)</td>
<td>3.92</td>
<td>3.41</td>
<td>-1.64</td>
<td>-2.71</td>
<td>6.64</td>
<td>2.24</td>
</tr>
<tr>
<td>VW returns (% p.a)</td>
<td>6.81</td>
<td>4.59</td>
<td>-0.16</td>
<td>-2.38</td>
<td>9.20</td>
<td>2.07</td>
</tr>
<tr>
<td>MV (£m)</td>
<td>956</td>
<td>685</td>
<td>1006</td>
<td>1628</td>
<td>-671</td>
<td>-14.55</td>
</tr>
<tr>
<td>CAPM Beta</td>
<td>0.81</td>
<td>0.90</td>
<td>0.94</td>
<td>1.08</td>
<td>-0.28</td>
<td>-8.60</td>
</tr>
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</table>
### Characteristics of decile Cokurtosis portfolios (From Negative CKT to Positive CKT)

<table>
<thead>
<tr>
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<th>P1</th>
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<th>P9</th>
<th>P10</th>
<th>P10-P1</th>
<th>t-stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average CKT</td>
<td>-0.76</td>
<td>-0.43</td>
<td>0.43</td>
<td>0.75</td>
<td>1.52</td>
<td>21.11</td>
</tr>
<tr>
<td>EW returns (% p.a)</td>
<td>-4.26</td>
<td>-2.17</td>
<td>2.41</td>
<td>3.36</td>
<td>7.63</td>
<td>3.00</td>
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<tr>
<td>VW returns (% p.a)</td>
<td>-0.58</td>
<td>-0.01</td>
<td>3.29</td>
<td>7.05</td>
<td>7.63</td>
<td>1.87</td>
</tr>
<tr>
<td>MV (£m)</td>
<td>1101</td>
<td>771</td>
<td>1056</td>
<td>1381</td>
<td>279</td>
<td>4.17</td>
</tr>
<tr>
<td>CAPM Beta</td>
<td>1.08</td>
<td>1.09</td>
<td>0.97</td>
<td>0.80</td>
<td>-0.28</td>
<td>-11.96</td>
</tr>
</tbody>
</table>
Performance of coskewness-sorted portfolios

<table>
<thead>
<tr>
<th></th>
<th>P1</th>
<th>P2</th>
<th>P9</th>
<th>P10</th>
<th>P1-P10</th>
<th>Chi-sq.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>CAPM alpha (% p.a)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.60</td>
<td>2.13</td>
<td>-2.74</td>
<td>-5.36</td>
<td>9.96</td>
<td></td>
<td>17.44</td>
</tr>
<tr>
<td>(1.54)</td>
<td>(0.71)</td>
<td>(-1.17)</td>
<td>(-2.07)**</td>
<td>(2.67)***</td>
<td>(0.06)</td>
<td></td>
</tr>
<tr>
<td><strong>Fama-French alpha</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.88</td>
<td>2.18</td>
<td>-2.83</td>
<td>-5.92</td>
<td>9.81</td>
<td></td>
<td>19.54</td>
</tr>
<tr>
<td>(1.57)</td>
<td>(0.79)</td>
<td>(-1.20)</td>
<td>(-2.41)**</td>
<td>(2.77)***</td>
<td>(0.03)</td>
<td></td>
</tr>
<tr>
<td><strong>Carhart alpha</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.37</td>
<td>2.70</td>
<td>-2.49</td>
<td>-3.38</td>
<td>7.74</td>
<td></td>
<td>22.00</td>
</tr>
<tr>
<td>(1.78)*</td>
<td>(1.09)</td>
<td>(-1.20)</td>
<td>(-1.17)</td>
<td>(2.07)**</td>
<td>(0.02)</td>
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</tbody>
</table>
### Performance of cokurtosis-sorted portfolios

<table>
<thead>
<tr>
<th>Alphas of value-weighted Cokurtosis portfolios</th>
<th>P1</th>
<th>P2</th>
<th>P9</th>
<th>P10</th>
<th>P10-P1</th>
<th>Chi-sq.</th>
</tr>
</thead>
<tbody>
<tr>
<td>CAPM alpha (% p.a)</td>
<td>-3.55</td>
<td>-3.01</td>
<td>0.62</td>
<td>4.86</td>
<td>8.41</td>
<td>19.34</td>
</tr>
<tr>
<td></td>
<td>(-1.15)</td>
<td>(-1.09)</td>
<td>(0.21)</td>
<td>(2.10)**</td>
<td>(2.10)**</td>
<td>(0.03)</td>
</tr>
<tr>
<td>Fama-French alpha</td>
<td>-4.52</td>
<td>-3.14</td>
<td>0.50</td>
<td>4.09</td>
<td>8.61</td>
<td>31.43</td>
</tr>
<tr>
<td></td>
<td>(-1.61)*</td>
<td>(-1.35)</td>
<td>(0.16)</td>
<td>(1.89)**</td>
<td>(2.32)**</td>
<td>(0.00)</td>
</tr>
<tr>
<td>Carhart alpha</td>
<td>-2.37</td>
<td>-2.74</td>
<td>0.82</td>
<td>5.19</td>
<td>7.57</td>
<td>36.69</td>
</tr>
<tr>
<td></td>
<td>(-0.78)</td>
<td>(-1.08)</td>
<td>(0.29)</td>
<td>(2.49)**</td>
<td>(1.96)**</td>
<td>(0.00)</td>
</tr>
</tbody>
</table>
Cross-sectional tests

- 2-step Fama-MacBeth tests for the cross-section of CSK (CKT) portfolios using standard market, size, value and momentum factors

- Main finding: Standard common factors cannot price the cross-section of CSK (or CKT) portfolios’ returns

- Construct coskewness and cokurtosis zero-cost (spread) risk factors using a 20% cutoff percentage value

- Augment the CAPM with these two risk factors

\[ r_i^e = \alpha_i + \beta_{i,m} r_m^e + \beta_{i,s^-s^+} (S^- - S^+) + \beta_{i,K^+_K^-} (K^+ - K^-) + \epsilon_i \]

- and use augmented higher co-moments model for asset pricing tests

- Evidence that these risk factors can help explain the cross-section of CSK and CKT portfolios (verification)
Robustness checks and current research

- Equally weighted portfolios
- Different windows of observations for the estimation of CSK and CKT (48 & 72 months)
- Different cutoff percentage values for the construction of the risk factors
- Current research: Utilize the higher co-moments asset pricing model to re-examine the abnormal performance of "anomalies" in the UK

- Preliminary results: Pricing errors are reduced (but not eliminated). Most interesting: Reduction of momentum abnormal profits.
Higher co-moments are relatively neglected in empirical asset pricing

Ample evidence that there is life beyond mean and variance

Strong theoretical foundations in asset pricing and utility theory vs. purely empirical risk factors

Negative coskewness risk and positive cokurtosis risk are significantly priced in a large cross-section of LSE-listed shares

Promising area of research for international financial markets, cost of capital, performance evaluation, anomalies studies