

# One-Factor Asset Pricing

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# Presentation Outline

- Motivation and Related Literature
- Generalized Disappointment Aversion SDF
- Asset Pricing Tests
- Disappointment Events
- Robustness Checks
- Conclusions

- Long tradition in asset pricing linking premia to consumption risk
  - C-CAPM: aggregate consumption growth should be a sufficient statistic for expected returns (Breedon, 1979)
  - Main implication:  $Risk\ Premium \propto \gamma Cov(r_{i,t+1}, \Delta c_{t+1})$
  - However, aggregate consumption growth has proved too smooth to
    - generate the empirical equity premium (Mehra and Prescott, 1985)
    - explain the cross-section of equity premia
- implied Risk Aversion coefficient implausibly high (Rabin, 2000)
- Recent attempts to engineer a more volatile SDF via alternative definitions of consumption risk (Parker and Julliard, 2005, Yogo, 2006, Jagannathan and Wang, 2007, Savov, 2011) have had limited success

- Empirical asset pricing literature dominated by multi-factor models since Fama and French (1993)
- Fishing license: Merton's ICAPM and Ross's APT
- Factors are typically return-generated and *a-theoretical*
- Unsettled debate whether factors proxy macro risks or capture anomalies
- Proliferation of factors and factor models (3-factor FF, 4-factor FFC, 4-factor HXZ, 5-factor FF...)
- Serious data mining concerns (Harvey et al., 2016)

- Propose a *single-factor, consumption-based* asset pricing model to explain the cross-section of equity returns
- Sole factor is an indicator of consumption growth being less than its certainty equivalent, derived from GDA preferences
- Intuitive interpretation of "bad times" and consumption risk
- Very good and robust empirical fit for various portfolio sorts and frequencies, comparable to Fama-French multi-factor models
- Plausible Disappointment Aversion coefficients

# Disappointment Aversion

- Model is based on Disappointment Aversion preferences (Gul, 1991)
- Axiomatic framework, but relaxing the independence axiom of EU
- *Asymmetric* treatment of losses vs gains à la Kahneman and Tversky (1979) & Benartzi and Thaler (1995) → kink in the utility function
- *But* reference point is endogenously defined (certainty equivalent of lottery) rather than imposed ad hoc
- Can resolve Allais paradox and explain Samuelson's famous gamble
- Investor exhibits *first-order risk aversion* (Segal and Spivak, 1990) unlike the traditional second-order risk aversion framework

# Generalized Disappointment Aversion

- Functional form for GDA preferences (Routledge and Zin, 2010):

$$u(\mu(p)) = \sum_{x_i \in X} p(x_i) u(x_i) - \theta \sum_{x_i \leq \delta \mu(p)} p(x_i) (u(\delta \mu(p)) - u(x_i))$$

where  $\mu(p)$  is the certainty equivalent for lottery  $p$ ,  $\theta$  is the DA coefficient, and  $\delta$  is the multiplier of  $\mu(p)$

- Interpretation: Investor imposes a penalty proportional to  $\theta$  on lottery outcomes below the disappointment threshold
- Can be combined with separable or non-separable utility functions

- Portfolio Choice:
  - Ang, Bekaert and Liu (2005)
  - Khanapure (2012)
  - Dahlquist, Faragó and Tédongap (2017)
- Equity premium:
  - Epstein and Zin (2001)
  - Routledge and Zin (2010)
  - Bonomo, Garcia, Meddahi and Tédongap (2011)
  - Dolmas (2014)
- Cross-sectional asset pricing:
  - Ostrovnaya, Routledge and Zin (2006)
  - Faragó and Tédongap (2017)
  - Delikouras (2017)



- Starting point: GDA intertemporal SDF of Routledge and Zin (2010)

$$M_{t+1}^{GDA} = \underbrace{\beta \left( \frac{C_{t+1}}{C_t} \right)^{\rho-1}}_{\text{C-CAPM}} \underbrace{\left[ \frac{V_{t+1}}{\mu_t(V_{t+1})} \right]^{\alpha-\rho}}_{\text{Epstein-Zin terms}} \underbrace{\left[ \frac{1 + \theta \mathbf{1}\{V_{t+1} \leq \delta \mu_t\}}{1 + \theta \delta^\alpha \mathbb{E}_t[\mathbf{1}\{V_{t+1} \leq \delta \mu_t\}]} \right]}_{\text{Disappointment Aversion correction}}$$

- $\mu_t(V_{t+1})$  : Certainty Equivalent of Lifetime Utility  $V_{t+1}$
- $\alpha$  : Risk Aversion coefficient,  $1/(1 - \rho)$  : EIS,  $\delta$  : multiplier of  $\mu_t$
- $\theta$  : Disappointment Aversion coefficient (overweigh losses)
- $\mathbf{1}\{V_{t+1} \leq \delta \mu_t\}$  : Indicator of disappointment events
- Nested preferences:
  - $\theta = 0 \rightarrow$  Epstein-Zin preferences
  - $\theta = 0 \ \& \ \alpha = \rho \rightarrow$  Power utility preferences

- ① Homoscedastic AR(1) consumption growth with normal shocks
- ② Disappointment Aversion *only* ( $\alpha = \rho = 1$ )
- Derive GDA-I SDF for *excess returns*:

$$M_{t+1}^{GDA-I} \propto \theta \mathbf{1} \left\{ \underbrace{\Delta c_{t+1} \leq \underbrace{\mu_c (1 - \phi_c) + \phi_c \Delta c_t}_{\text{certainty equivalent}} + \underbrace{d_2 \sqrt{1 - \phi_c^2 \sigma_c}}_{\text{st dev}}}_{\text{conditional mean}} \right\}$$

- Threshold for disappointment event depends on deviation from expected consumption growth controlled by  $d_2$
- Consumption growth covariance risk is not priced
- GDA-I is an indicator function of consumption growth being less than its certainty equivalent  $\rightarrow$  *bi-modal SDF*

# Comparison with Popular SDFs

- C-CAPM SDF:

$$M_{t+1}^{C-CAPM} \propto -\gamma \Delta c_{t+1}$$

- CAPM SDF:

$$M_{t+1}^{CAPM} \propto -b_m R_{m,t+1}^x$$

- NBER SDF:

$$M_{t+1}^{NBER} \propto \lambda \mathbf{1}\{> 4 \text{ NBER recession months in a year}\}$$

- Fama-French 3-factor SDF:

$$M_{t+1}^{FF3} \propto -b_m R_{m,t+1}^x - b_{SMB} R_{SMB,t+1} - b_{HML} R_{HML,t+1}$$

- Fama-French 5-factor SDF:

$$M_{t+1}^{FF5} \propto -b_m R_{m,t+1}^x - b_{SMB} R_{SMB,t+1} - b_{HML} R_{HML,t+1} - b_{RMW} R_{RMW,t+1} - b_{CMA} R_{CMA,t+1}$$

- Sample period: 1933-2012
- BEA consumption of services and non-durables
- Normalized by population and PCE price index
- NBER recession periods
- K. French's online data library
  - 6 & 25 & 100 Size/BM portfolios
  - 25 Size/OP & 25 Size/INV portfolios (post '64)
  - 10 LTR portfolios
  - 10 E/P portfolios (post '53)
  - Fama-French factors
- Corporate bond and equity index option portfolios

- Need to specify consumption growth moments to identify disappointment events and test GDA-I model
- Fit the empirical consumption growth moments *jointly* with Euler equations for excess portfolio returns for each cross-section via GMM:

$$\begin{bmatrix} \mathbb{E}[\Delta c_t] - \mu_c \\ \mathbb{E}[\Delta c_t^2] - \mu_c^2 - \sigma_c^2 \\ \mathbb{E}[\Delta c_t \Delta c_{t-1}] - \mu_c^2 - \phi_c \sigma_c^2 \\ \mathbb{E}[(R_{i,t} - R_{1y,t}) (1 - \mathbb{E}[M_t^{GDA-I}] + M_t^{GDA-I})] \end{bmatrix} = \mathbf{0}$$

- Weighting matrix ensures consumption growth moments matching
- Competing asset pricing models estimated via GMM using identity weighting matrix

# Annual 25 Size/BM Portfolios: Estimates

	<b>GDA-I</b>	<b>CCAPM</b>	<b>CAPM</b>	<b>FF3</b>	<b>FF5</b>	<b>NBER</b>
<b>GDA ind</b>	<b>4.126</b> [4.207]					
<b>d<sub>2</sub></b>	<b>-0.77</b> [-3.006]					
<b>CONS</b>		<b>57.331</b> [3.499]				
<b>MKT</b>			<b>2.935</b> [4.781]	<b>2.043</b> [2.375]	<b>3.191</b> [2.218]	
<b>SMB</b>				<b>0.335</b> [0.309]	<b>1.912</b> [1.242]	
<b>HML</b>				<b>3.026</b> [3.079]	<b>-0.473</b> [-0.153]	
<b>RMW</b>					<b>0.967</b> [0.308]	
<b>CMA</b>					<b>9.321</b> [2.080]	
<b>NBER ind</b>						<b>9.157</b> [1.053]
<b><math>\chi^2</math></b>	<b>28.795</b>	<b>87.58</b>	<b>97.94</b>	<b>62.46</b>	<b>42.57</b>	<b>5.543</b>
<b>dof</b>	<b>23</b>	<b>24</b>	<b>24</b>	<b>22</b>	<b>20</b>	<b>24</b>
<b>p</b>	<b>0.187</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0.002</b>	<b>1</b>
<b>RMSE</b>	<b>1.345</b>	<b>2.107</b>	<b>2.973</b>	<b>1.648</b>	<b>1.563</b>	<b>2.358</b>
<b>R<sup>2</sup></b>	<b>0.901</b>	<b>0.758</b>	<b>0.519</b>	<b>0.852</b>	<b>0.821</b>	<b>0.697</b>

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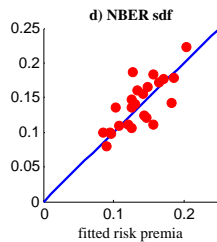
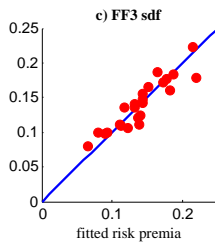
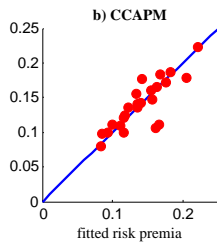
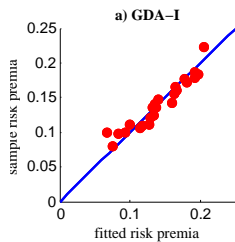
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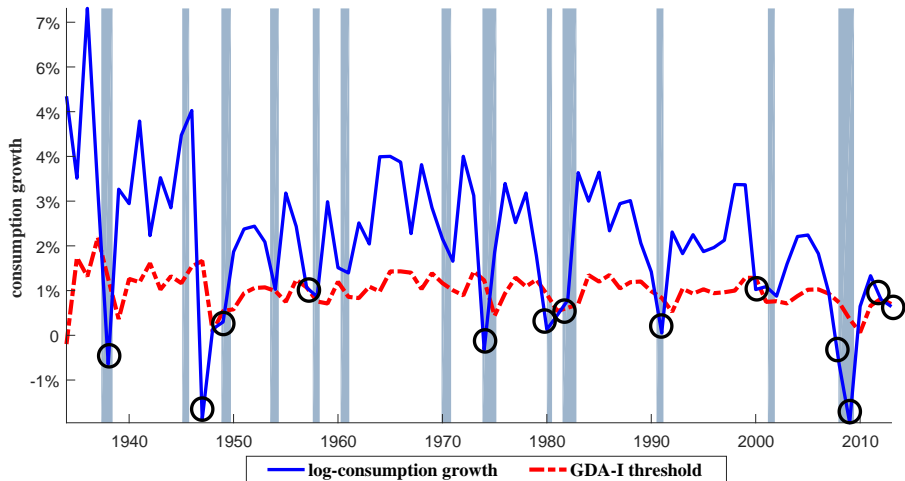


# Annual 25 Size/BM Portfolios: Fit



# Disappointment Years

{1937, '46, '48, '56, '73, '79-80, '90, '99, 2007-08, 2011-12}



# Disappointment Years' Characteristics

	<u>Full Sample</u>	<u>Disappointment Yrs</u>	<u>Non-Disappointment Yrs</u>
<b>Market Premium</b>	9.16%	-3.05%	11.52%
<b>SMB Premium</b>	2.12%	-5.68%	3.63%
<b>HML Premium</b>	9.02%	4.23%	9.95%
<b>S&amp;P 500 Daily St. Dev.</b>	15.43%	19.22%	14.70%
<b>Term spread</b>	1.62%	0.99%	1.74%
<b>Real Consumption growth</b>	2.21%	-0.07%	2.65%
<b>Earnings growth</b>	11.23%	3.20%	12.78%
<b>Net Equity Expansion</b>	1.52%	0.77%	1.66%
<b>cay</b>	0.00%	-0.16%	0.03%
<b>Change in unemployment, t+1</b>	0.06%	0.97%	-0.10%
<b><math>\Delta\%</math> in Consumer Confidence</b>	0.16%	-7.49%	1.80%

# Monthly 25 Size/BM Portfolios: Estimates

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<b>GDA ind</b>	<b>3.673</b> [3.293]					
<b>CONS</b>		<b>248.151</b> [2.464]				
<b>MKT</b>			<b>3.579</b> [4.974]	<b>2.342</b> [2.847]	<b>3.625</b> [2.245]	
<b>SMB</b>				<b>0.517</b> [0.434]	<b>6.724</b> [3.638]	
<b>HML</b>				<b>4.905</b> [4.521]	<b>2.138</b> [0.367]	
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$\chi^2$	<b>41.296</b>	<b>70.582</b>	<b>121.875</b>	<b>94.856</b>	<b>94.631</b>	<b>9.662</b>
dof	<b>24</b>	<b>24</b>	<b>24</b>	<b>22</b>	<b>20</b>	<b>24</b>
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RMSE	<b>0.119</b>	<b>0.283</b>	<b>0.249</b>	<b>0.156</b>	<b>0.142</b>	<b>0.174</b>
R <sup>2</sup>	<b>0.818</b>	<b>-0.496</b>	<b>0.178</b>	<b>0.677</b>	<b>0.673</b>	<b>0.601</b>

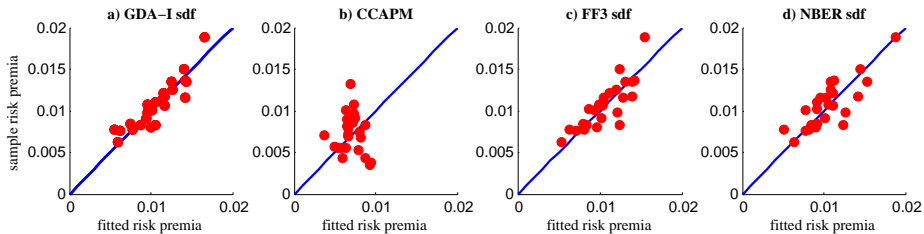
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# Monthly 25 Size/BM Portfolios: Fit



# Why does GDA-I SDF work?

- GDA-I SDF implies that an asset's **premium** should be a **linear function** of its **losses** during **disappointment events**
- The standard asset pricing equation

$$\mathbb{E}[R_{i,t} - R_{f,t}] = -\text{Cov}(R_{i,t} - R_{f,t}, M_t)$$

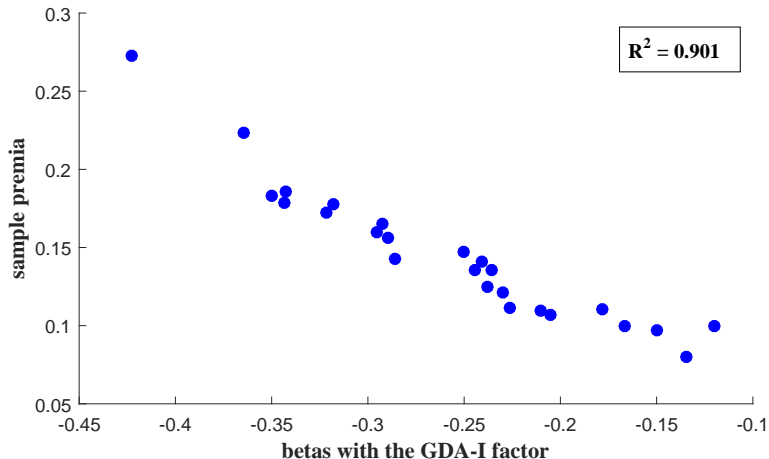
yields for GDA-I SDF:

$$\mathbb{E}[R_{i,t} - R_{f,t}] = -\frac{\theta \mathbb{E}[\mathbf{1}_t]}{1 - \theta \mathbb{E}[\mathbf{1}_t]} \mathbb{E}[(R_{i,t} - R_{f,t}) | \mathbf{1}_t = 1]$$

- GDA-I SDF successfully aligns portfolio premia with their *disappointment betas*



# Premia & Disappointment Betas: 25 Size/BM Portfolios



GDA-I SDF performs very well with:

- 1 Other equity cross-sections (Size/OP, Size/INV, LTR, E/P, Joint)
- 2 Great Depression, pre-crisis & post-WWII periods
- 3 Recursive estimation of disappointment events
- 4 Value-weighted portfolios
- 5 Quarterly data
- 6 Two-stage GMM estimation
- 7 Other asset classes (corporate bond & equity index option portfolios)

## 1 Weak Identification?

Disappointment betas are statistically significant and exhibit substantial cross-sectional dispersion

## 2 Sensitivity to Disappointment Threshold?

Explanatory power remains strong even if we use  $\tilde{d}_2 = \hat{d}_2 \pm std\ error(\hat{d}_2)$  to determine new disappointment threshold & events

## 3 Spurious Explanatory Power?

1m *placebo* indicators confirm that the success of GDA-I SDF is not spuriously driven by its functional form

# Conclusions

- Propose a simple SDF to explain the cross-section of equity returns
- GDA-I SDF consists of an indicator function of consumption growth being less than its certainty equivalent
- Founded on (G)DA preferences of Gul & Routledge and Zin
- Yields very good empirical fit and plausible DA coefficients for various cross-sections
- Question the ability of second-order risk aversion (smooth utility functions) to explain the cross-section of equity returns
- Support consumption-based asset pricing but highlight the importance of downside consumption risk